Detection Performance from Compressed Measurements

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Outline

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Summary

This work compares the performance of the **four different estimators:**

1. Compressed matched filter
2. Soft-thresholded compressed matched filter
3. ℓ_1-regularized least squares
4. Complex approximate message passing (CAMP)

We do so over a wide range of signal-to-noise ratio and under-sampling factor using **two different performance metrics:**

1. Scene reconstruction error
2. Target detection probability

Radar System Detection

Common radar application: monitor the locations of targets within some field of regard.

- Path of the signal includes:
  1. Antenna
  2. RF electronics
  3. Analog and digital signal processing
  4. Detector
- The resulting detections are sent to a multi-target tracker, fire control processor, or other higher-level data processor [Richards, 2005]

Prior Work on CS and Radar

Compressed sensing (CS) techniques seem well-suited to this task of radar signal processing and target detection:

- Small number of targets relative to possible target locations
- Fine range resolution the radar system requires wide waveform bandwidths: challenging to sample at the Nyquist rate.

Prior work on the topic has shown promise as well:

- Several authors have noted these and other advantages that could be gained by such an approach [Strohmer and Friedlander, 2011], [Herman and Strohmer, 2009], [Chen and Vaidyanathan, 2008].
- Others have described techniques that could be used in a CS radar implementation [Tropp et al., 2010], [Baransky et al., 2012].

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Fully-Sampled Measurements

Define the sensing problem as follows:

- The scene is described by $\mathbf{x} \in \mathbb{C}^n$ with only s non-zero components at locations $\mathcal{S}$ that represent the targets in that scene.
- The radar probes that scene using some transmitted waveform and makes measurements of the returning signal at the Nyquist rate. Let this process be modeled by the matrix $\mathbf{S} \in \mathbb{C}^{n \times n}$.
- However the measurements are corrupted by zero-mean circular complex noise, $\mathbf{n} \sim \mathcal{CN}(0, \frac{1}{\sqrt{SNR}})$.

The measurements are then:

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{n}.$$

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Compressed Measurements

The compressed measurements $\mathbf{z} \in \mathbb{C}^m$ are made by applying some compression operator, $\mathbf{C} \in \mathbb{C}^{m \times n}$, to the fully-sampled data. We model this as

$$\mathbf{z} = \mathbf{C}\mathbf{y}.$$

- The combined sensing operator $\mathbf{A} = \mathbf{C}\mathbf{S} \in \mathbb{C}^{m \times n}$.
- Call the ratio n/m the under-sampling factor (USF).

Tested Parameters

Want to choose sensing matrices $\mathbf{C}$ and $\mathbf{S}$ that exhibit very low mutual coherence and allow the CS algorithms to work within design assumptions.

- Entries in $\mathbf{C}$ be independent and identically distributed (IID), taking on the values $\pm\frac{1}{\sqrt{n}}$ with equal probability
- Entries in $\mathbf{S}$ be IID taking the values $\pm\frac{1}{\sqrt{m}}$ with equal probability

Additionally, let

- $n = 2000$
- $s = 4$, these 4 targets are placed randomly in the scene and have unit amplitude and zero phase

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Matched Filter Estimate

The matched filter estimator is calculated by applying the conjugate transpose of the complete sensing operator

$$\hat{\mathbf{x}}_{MF} = \mathbf{A}^H \mathbf{z}.$$

Matched Filter + Soft Thresholding Estimate

Define the soft thresholding function as

$$g(\mathbf{h}, \rho) = \begin{cases} \mathbf{h} - \rho \frac{\mathbf{h}}{|\mathbf{h}|}, & |\mathbf{h}| > \rho \\ 0, & \text{otherwise} \end{cases}$$

and let ρ^* be the 90th percentile of the entries in $|\hat{\mathbf{x}}_{MF}|$. Then the matched filter-soft thresholding solution is

$$\hat{\mathbf{x}}_{MFST} = g(\hat{\mathbf{x}}_{MF}, \rho^*).$$

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ℓ_1-Regularized Least Squares Estimate

The ℓ_1-regularized least squares estimate is the one that solves the following inequality:

$$\hat{\mathbf{x}}_{CS} = \arg\min_{\mathbf{x}} \left\| \mathbf{z} - \mathbf{A}\mathbf{x} \right\|_2^2 + \lambda \left\| \mathbf{x} \right\|_1 .$$

We solve this using the TFOC package [Becker et al., 2012] with $\lambda = .7$.

CAMP Estimate

Finally we compute the CAMP estimate using our own implementation of the algorithm described in [Anitori et al., 2013], [Maleki et al., 2013].

- Iterative algorithm finds a solution that balances fidelity to the measured data with a sparsity condition.
- The code is structured to produce two outputs
  1. A sparse target component plus white noise.
  2. A soft-thresholded version of that noisy solution that leaves (ideally) just the sparse component.
- We compute both solutions but only show the first one because it provides better detection performance.

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Testing Methodology

We performed the following analysis:

- Establish a grid of (SNR, USF) pairs
  - SNR ranging from 0 to 40 dB
  - USF ranging from 0 to 50
- At each point in the grid perform 100 trials in which new random target scene, noise vector, and sensing matrices are generated and the target scene is estimated using the four estimate techniques described
- Calculate average performance at each point

Performance Metrics: Norm Reconstruction Error

And, independent of any detection thresholds or criteria, the **norm reconstruction error** is defined as

$$E_{norm} = \frac{||\mathbf{x} - \hat{\mathbf{x}}||_2}{||\mathbf{x}||_2}.$$

Performance Metrics: Probability of Detection

For a given probability of false alarm, P_{FA}, a **detection threshold** can be calculated:

- The true scene $\mathbf{x}$ that has n elements and s targets at locations specified by $\mathcal{S}$ and an estimate of that scene $\hat{\mathbf{x}}$
- And $\hat{\mathbf{x}}_s$ contains all the values of $|\hat{\mathbf{x}}|$ sorted in increasing order then the detection threshold $D = \hat{\mathbf{x}}_s(\lceil P_{FA} n \rceil)$

Using this detection threshold the **probability of detection** is the fraction of elements of the estimate at the true target locations that have magnitude greater than the threshold:

$$P_D = \mathrm{frac}\left(|\hat{\mathbf{x}}(\mathcal{S})| > D\right)$$

Norm Error Matrix

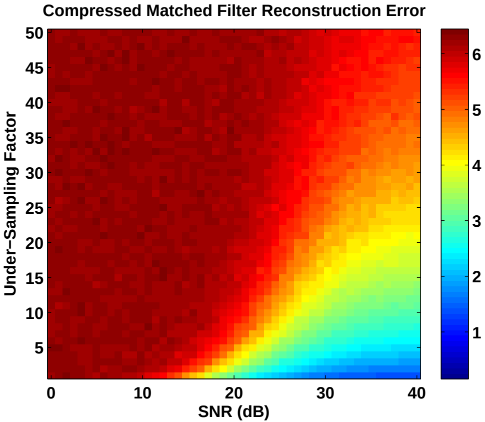

Figure: Matched filter reconstruction error (E_{norm}) is shown over the input parameter space. Blue indicates better performance.

Norm Error Frontiers

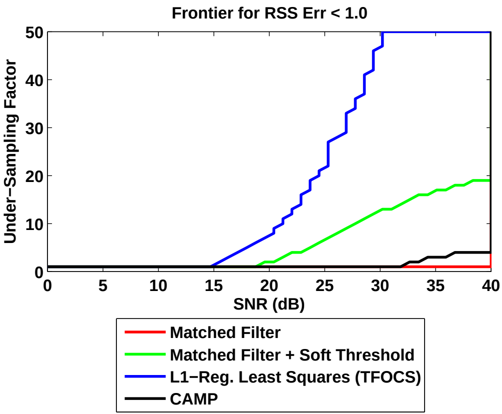

Figure: A comparison of reconstruction error (E_{norm}) performance frontiers shows that the sparsity-favoring techniques achieve better reconstruction error of this sparse signal. Curves that lie higher and further to the left indicate better performance.

Probability of Detection Matrix

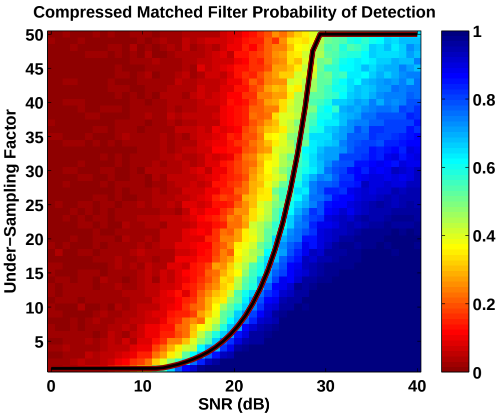

Figure: Matched filter probability of detection (P_D) is shown over the input parameter space. Blue indicates better performance. This plot uses $P_{FA} = .01$. The black line superimposed on the graph shows the theoretical boundary for a $P_D = .9$.

Probability of Detection Frontiers

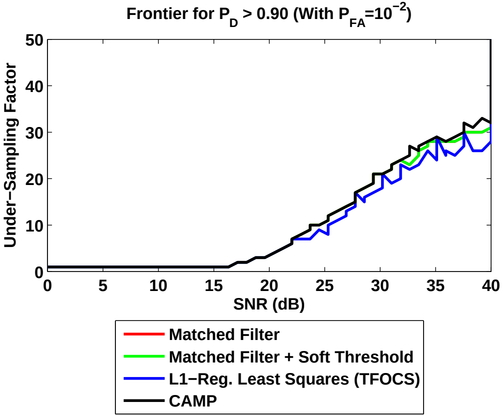

Figure: A comparison of detection (P_D) performance frontiers shows that all these techniques achieve approximately equal detection performance. Curves that lie higher and further to the left indicate better performance.

Execution Time

Estimator	Mean Duration (s)	Standard Deviation (s)
Matched Filter	3.0×10^{-3}	3.0×10^{-3}
Matched Filter + Soft Threshold	3.3×10^{-3}	3.0×10^{-3}
ℓ_1-Regularized Least Squares	6.4×10^{-2}	5.4×10^{-2}
CAMP	3.0	4.3

Table: Summary statistics for execution time of the tested algorithms over all input parameter combinations are given.

Theoretical Detection Performance

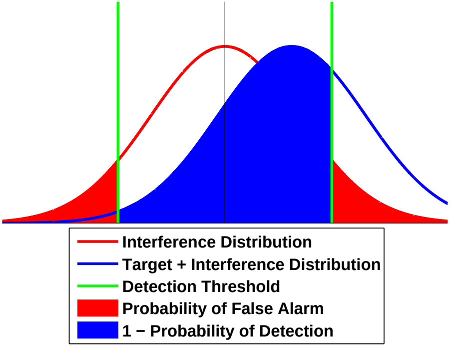

Figure: This plot illustrates the relationship between probability of detection and probability of false alarm for a deterministic signal in Gaussian interference.

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Conclusion

This work:

- Evaluates the ability of different algorithms to detect targets from compressive measurements over a **wide range** of SNR and USF
- Shows that the sparsity-favoring solutions produce results with better **norm error**
- Shows that the matched filter performs as well as more computationally expensive algorithms at **detecting targets**
- Encourages researchers to report results in terms of detection statistics
- Points to **future work** to
  - Express the random variable distributions for target present / absent for the compressed matched filter estimator and thereby calculate theoretical P_D and P_{FA}
  - Derive CS recovery algorithm thresholds in terms of P_D and P_{FA}

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