Detection Performance from Compressed Measurements

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***Abstract*—This work uses two performance metrics, target detection and scene reconstruction performance, to compare various estimation techniques that operate on compressed measurements. Specifically we compare the performance of the compressed matched filter, ℓ_1-regularized least squares, and complex approximate message passing (CAMP), as well as a sparsified matched filter estimate. We show that the compressed matched filter provides the same or similar detection performance as the other, more computationally expensive techniques, but at the expense of poorer signal reconstruction error. However, by sparsifying the matched filter estimate using a soft-thresholding function, this estimate can achieve high reconstruction performance as well, and at much lower computational cost.**

***Index Terms*—Target Detection; Compressed Sensing**

I. Introduction

Radar and similar remote sensing technologies are often used to monitor the locations of targets within some field of regard. In a common radar system design the signal is gathered by the antenna, amplified by some radio frequency electronics, processed by an analog and digital signal processing chain, and finally converted into a discrete set of detections by an algorithm that judges the presence or absence of a target at each sample. An adaptive threshold will often be used to approach a constant false alarm rate (CFAR). The resulting detections are sent to a multi-target tracker, fire control processor, or other higher-level data processor [1].

Compressed sensing (CS) techniques seem well-suited to this task of radar signal processing and target detection. Typically the number of targets is much smaller than the number of possible target locations which introduces a natural sparsity. And to achieve fine range resolution the radar system may utilize wide waveform bandwidths that are challenging to sample at the Nyquist rate. Several authors have noted these and other advantages that could be gained by such an approach [2], [3], [4]. Others have described techniques that could be used in a CS radar implementation [5], [6].

Once such compressed measurements are acquired many techniques can be employed to solve for the locations and amplitudes of the targets. Templates for First-Order Convex Solvers (TFOCS) is a convex optimization solution framework that accommodates expressions like those that are frequently generated in CS [7]. And Complex Approximate Message Passing (CAMP) is an extension of successful AMP algorithms to the complex domain found in radar signal processing. Anitori, et al, made use of the fact that the interference in the CAMP estimate is normally distributed to show a compressed sensing CFAR algorithm [8].

Additionally, the matched filter, though commonly used in the fully-sampled setting, also maximizes the signal-to-noise ratio (SNR) of the estimate from compressed measurements. And the computation complexity of the matched filter is significantly lower than the aforementioned CS estimators. We will compare these various estimation techniques with particular emphasis on detection performance.

II. Method

A. Problem Formulation

Let the scene be a vector $\mathbf{x} \in \mathbb{C}^n$ with only s non-zero components that represent the targets in that scene. The radar probes that scene using some transmitted waveform and makes measurements of the returning signal at the Nyquist rate. Let this process be modeled by the matrix $\mathbf{S} \in \mathbb{C}^{n\times n}$. However the measurements are corrupted by zero-mean circular complex noise, $\mathbf{n} \sim \mathcal{CN}(0, \frac{1}{\sqrt{SNR}})$. The measurements are then

$$\mathbf{y} = \mathbf{S}\mathbf{x} + \mathbf{n}.$$

The compressed measurements $\mathbf{z} \in \mathbb{C}^m$ are made by applying some compression operator, $\mathbf{C} \in \mathbb{C}^{m\times n}$, to the fully-sampled data. We model this as

$$\mathbf{z} = \mathbf{C}\mathbf{y}.$$

The combined sensing operator $\mathbf{A} = \mathbf{CS} \in \mathbb{C}^{m\times n}$. Call the ratio n/m the under-sampling factor (USF).

B. Solution Techniques

The matched filter estimator is calculated by applying the conjugate transpose of the complete sensing operator

$$\hat{\mathbf{x}}_{MF} = \mathbf{A}^H\mathbf{z}.$$

Next, define the soft thresholding function as

$$g(\mathbf{h}, \rho) = \begin{cases} \mathbf{h} - \rho\frac{\mathbf{h}}{|\mathbf{h}|}, & |\mathbf{h}| > \rho \\ 0, & \text{otherwise} \end{cases}$$

and let ρ^* be the 90th percentile of the entries in $|\hat{\mathbf{x}}_{MF}|$. Then the matched filter-soft thresholding solution is

$$\hat{\mathbf{x}}_{MFST} = g(\hat{\mathbf{x}}_{MF}, \rho^*).$$

The ℓ_1-regularized least squares estimate is the one that solves the following inequality:

$$\hat{\mathbf{x}}_{cs} = \arg\min_{\mathbf{x}} ||\mathbf{z} - \mathbf{CSx}||_2^2 + \tau \, ||\mathbf{x}||_1 \, .$$

We solve this using the TFOC package [9].

Finally we compute the CAMP estimate using our own implementation of the algorithm described in [8], [10]. This iterative algorithm also finds a solution that balances fidelity to the measured data with a sparsity condition. The code is structured to produce two outputs. One solution consists of a sparse target component plus white noise. The second solution is a soft-thresholded version of that noisy solution that leaves (ideally) just the sparse component. We compute both solutions but only show the first one because it provides better detection performance.

C. Evaluation Criteria

In an operational radar system, the statistics of the interference would be estimated to set a detection threshold. The CFAR detector uses a window around the cell under test to estimate those statistics locally. However, to reduce the dependence of our results on this local estimation we use the following oracle-like detection criteria.

For a given probability of false alarm, P_{FA}, a detection threshold can be calculated. Assume a true scene $\mathbf{x}$ that has n elements and s targets at locations specified by $\mathcal{S}$ and an estimate of that scene $\hat{\mathbf{x}}$. If $\hat{\mathbf{x}}_s$ contains all the values of $|\hat{\mathbf{x}}|$ softed in increasing order then the detection threshold $D = \hat{\mathbf{x}}_s(\lceil P_{FA} n \rceil)$. Using this detection threshold the probability of detection is the fraction of elements of the estimate at the true target locations that have magnitude greater than the threshold:

$$P_D = \text{frac}\left(|\hat{\mathbf{x}}(\mathcal{S})| > D\right).$$

And, independent of any detection thresholds or criteria, the RSS reconstruction error is defined as

$$E_{RSS} = \frac{||\mathbf{x} - \hat{\mathbf{x}}||_2}{||\mathbf{x}||_2}.$$

D. Tested Parameters

To specify the problem let entries in $\mathbf{C}$ be independent and identically distributed (IID), taking on the values $\pm\frac{1}{\sqrt{n}}$ with equal probability. Similarly let entries in $\mathbf{S}$ be IID taking the values $\pm\frac{1}{\sqrt{m}}$ with equal probability. These sensing matrices exhibit very low mutual coherence and allow the CS algorithms to work within design assumptions. And let $n = 2000$ and $s = 4$. These 4 targets are placed randomly in the scene and have unit amplitude and zero phase.

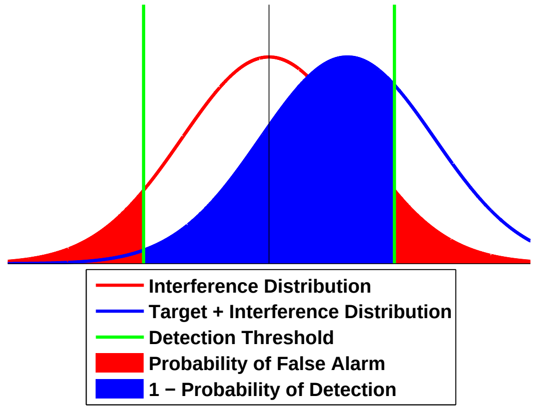

Fig. 1: This plot illustrates the relationship between probability of detection and probability of false alarm for a deterministic signal in Gaussian interference.

E. Detection Theoretic Performance

In detection theory a measurement consists of stochastic interference and perhaps some deterministic non-zero signal. Thus the goal is to develop a criterion to determine which of two hypotheses is obserbed: interference or interfernce plus signal. For a real signal in Gaussian noise, the relationship between SNR, P_D, and P_{FA} can be calculated using the definitions of the normal distribution. This calculation is illustrated in Figure 1.

III. Results

We performed 100 random trials of the described experiment at each (SNR, USF) pair and show the average results over those trials. Figures 2 and 3 show the average detection and reconstruction performance, respectively, of the matched filter solution over the span of tested parameters. The easiest problem posed is that in the bottom right corner where the most samples are taken and the signal is the strongest. Thus it is unsurprising that performance in this region is very good.

The results in Figure 2 agree rather well with the theoretical detection curve illustrated in but start to show difference for higher levels of undersampling. This curve assumes that the Gaussian interference whereas the interference in the estimate is not Gaussian, even though it may be approximated as such. Work in this area continues to rigourously characterize the detection behavior at higher levels of undersampling.

These results and similar results for the other solution techniques can be summarized by plotting the performance frontiers at which the estimate drops below some specified threshold. These frontiers are show in Figure 4 and 5. Notably, the detection performance of all the estimators is comparable. In contrast, the sparsity-favoring ℓ_1-regularized least squares solution gives a better reconstruction error. Note, also, that the simple addition of the soft-thresholding operator reduces the reconstruction error significantly, and a more aggressively applied threshold could reduce it further.

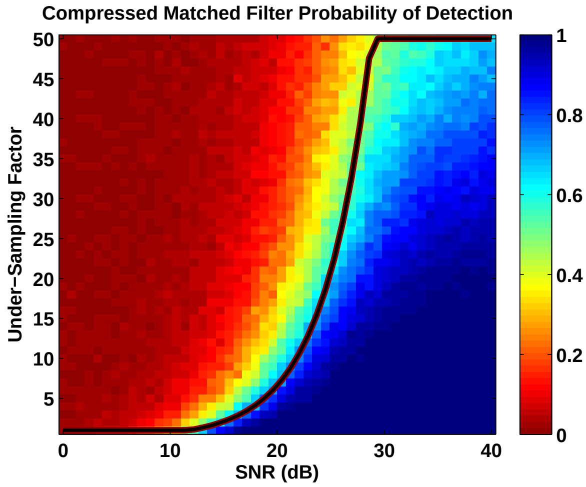

Fig. 2: Matched filter probability of detection (P_D) is shown over the input parameter space. Blue indicates better performance. This plot uses $P_{FA} = .01$. The black line superimposed on the graph shows the theoretical boundary for a $P_D = .9$.

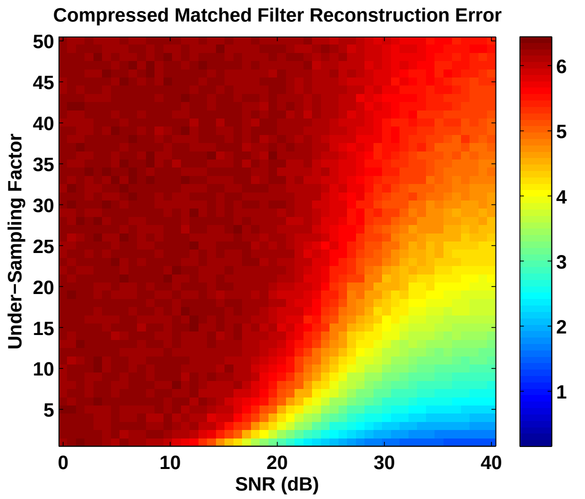

Fig. 3: Matched filter reconstruction error (E_{RSS}) is shown over the input parameter space. Blue indicates better performance.

Finally, we measure the average execution time of the reconstruction methods running in MATLAB on a desktop computer with an Intel Core i5 processor and 16 GB of memory. The matched filter and soft thresholded matched filter both execute quickly. The ℓ_1-regularized least squares (using the TFOCS implementation) executes around $20\times$ more slowly. And, our implementation of CAMP is much slower than either of those two. With a more-refined stopping criterion, this time could likely be reduced significantly.

IV. Conclusion

Although the use of sparsity-favoring solution techniques can produce substantial gains in the signal reconstruction error, the detection performance is largely a function of the SNR and

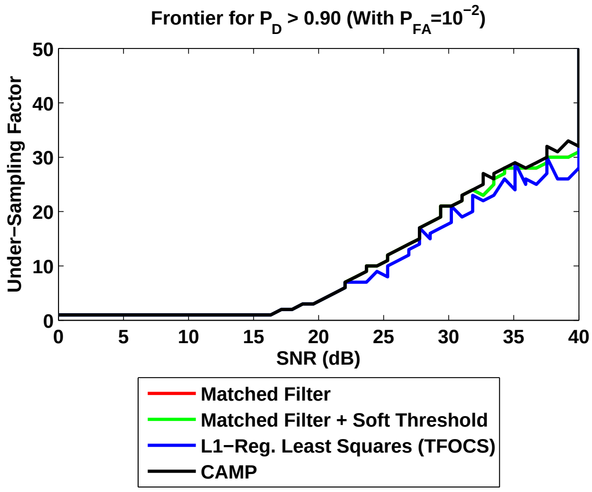

Fig. 4: A comparison of detection (P_D) performance frontiers shows that all these techniques achieve approximately equal detection performance. Curves that lie higher and further to the left indicate better performance. By this measure the matched filter and the sparsified matched filter perform identically, thus the red and the green curve lies directly on top of the red curve.

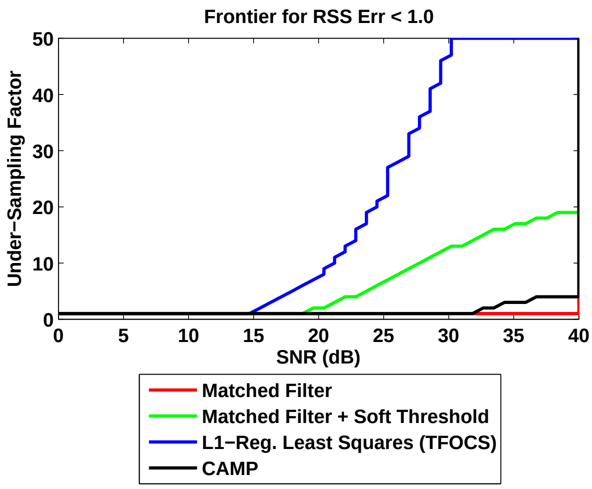

Fig. 5: A comparison of reconstruction error (E_{RSS}) performance frontiers shows that the sparsity-favoring techniques achieve better reconstruction error of this sparse signal. Curves that lie higher and further to the left indicate better performance.

Estimator	Mean Duration (s)	Standard Deviation (s)
Matched Filter	3.0×10^{-3}	3.0×10^{-3}
Matched Filter + Soft Threshold	3.3×10^{-3}	5.4×10^{-2}
ℓ_1-Regularized Least Squares	6.4×10^{-2}	3.0×10^{-3}
TFOCS	3.0	4.3

TABLE I: Summary statistics for execution time of the tested algorithms over all input parameter combinations are given.

USF and independent of solution technique. Future work will compare this detection performance to those derived predictions in [11], [12].

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