Compressed Sensing Radar Amid Noise and Clutter

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Outline

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Motivation

- Detecting surface targets from an airborne platform has wide utility for maintaining awareness of battlefield movements
- In many such airborne surveillance applications strong ground returns swamp target energy and cannot be neglected
- Unlike noise interference, clutter cannot be mitigated by a more energetic waveform
- Targets can be separated from the clutter returns based on structure of the clutter returns

Moving Target Indication (MTI) is an important and challenging problem for radar designers and signal processing engineers to solve

Pulse Doppler Radar

Airborne radar typically samples in three dimensions to build a data cube [Richards, 2005]

- **Fast Time:** Sample-to-sample spacing in the ADC: if the receiver uses both I and Q channels it is the inverse of the transmitted pulse bandwidth
- **Space:** Element-to-element spacing the antenna array: To avoid grating lobes this is smaller than $\lambda/2$
- **Slow Time:** Pulse-to-pulse spacing in the CPI: This spacing defines Doppler velocities that may be received unambiguous

These three dimensions of sampling form the measured data-cube

Problem Setup

Write the problem as

$$\mathbf{y} = \mathbf{y}_s + \mathbf{y}_i = (\mathbf{S}\mathbf{x}) + (\mathbf{S}\mathbf{c} + \mathbf{n}) = \mathbf{S}(\mathbf{x} + \mathbf{c}) + \mathbf{n}$$

The most common estimation technique is the matched filter which is computed by applying the adjoint operator

$$\hat{\mathbf{x}}_{adj} = \mathbf{S}^H \mathbf{y}$$

But this technique does not account for clutter

We set up the problem as a linear estimation problem to apply compressed sensing techniques

Space-Time Adaptive Processing

STAP techniques estimate the interference covariance matrix

$$\mathbf{R} = \mathrm{E}\left[\mathbf{y}_i \mathbf{y}_i^H\right]$$

Use that to generate a filter to minimize interference

$$\mathbf{W} = \mathbf{R}^{-1}\mathbf{S}$$

This filter maximizes SINR when applied to the measured data [Melvin, 2004][Guerci, 2003]

$$\hat{\mathbf{x}}_{stap} = \mathbf{W}^H \mathbf{y} = \mathbf{S}^H \mathbf{R}^{-1} \mathbf{y}$$

Space-time adaptive processing (STAP) is an ideal way to detect weak targets in the midst of strong clutter returns

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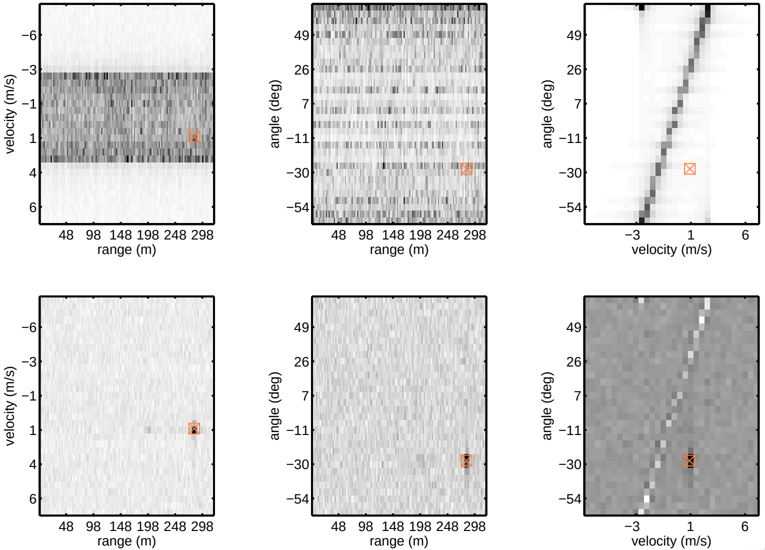
STAP Example



STAP Challenges

The interference covariance matrix must be estimated

$$\mathbf{R} = \mathrm{E}\left[\mathbf{y}_i \mathbf{y}_i^H\right]$$

- This is computed from training data in the current CPI
- But if a target is present in the training data, it will be filtered
- So a new estimate can be generated from measured cells surrounding the current cell (with some guard cells)

That estimated matrix must be inverted

$$\hat{\mathbf{x}}_{stap} = \mathbf{S}^H \mathbf{R}^{-1} \mathbf{y}$$

- Repeated matrix inversions can be computationally expensive
- Covariance matrix may be low-rank and therefore ill-posed for inversion

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Compressive Measurements

The standard radar system acquires measurements according to

$$\mathbf{y} = \mathbf{y}_s + \mathbf{y}_i = (\mathbf{Sx}) + (\mathbf{Sc} + \mathbf{n}) = \mathbf{S}(\mathbf{x} + \mathbf{c}) + \mathbf{n}$$

The compressive system acquires fewer measurements according to

$$\mathbf{z} = \mathbf{Cy} = \mathbf{CS}(\mathbf{x} + \mathbf{c}) + \mathbf{n}$$

- $\mathbf{C} \in \mathbb{C}^{m \times n}$ and $m < n$
- Under-sampling factor (USF) is n/m

This yields an ill-posed, underdetermined system of equations which theory predicts can be solved under two conditions [Candès and Romberg, 2007]:

1. The vector to be solved for is sparse
2. The compressive measurement vectors are incoherent with the sparsity basis

Proposed Extension: CA CS

A standard compressed sensing solution would solve

$$\hat{\mathbf{x}}_{cs} = \arg\min_{\mathbf{x}} \|\mathbf{z} - \mathbf{CSx}\|_2^2 + \tau \|\mathbf{x}\|_1$$

We propose a covariance-aware compressed sensing (CA CS):

$$\hat{\mathbf{x}}_{cacs} = \arg\min_{\mathbf{x}} (\mathbf{z} - \mathbf{CSx})^H \mathbf{R}_c^{-1} (\mathbf{z} - \mathbf{CSx}) + \gamma \|\mathbf{x}\|_1$$

- $\mathbf{R}_c = \mathbf{CRC}^H$

This problem can be solved by the same types of optimization routines as the more generic compressed sensing problem

We extend existing CS theory to account for the known covariance interference matrix.

Test Environment

- Adaptive Sensor Prototyping ENvironment (ASPEN), developed at GTRI/SEAL, is a flexible radar and clutter modeling tool used to generate test signals
- Developed $\mathbf{S}$, a linear model thereof, based on FFTs
  - For a problem of size $32 \times 32 \times 128$ the explicit $\mathbf{S}$ has 1.7×10^{10} element
- Simple $\mathbf{C}$ is iid ± 1
- Solved for $\hat{\mathbf{x}}_{cacs}$ using TFOCS [Becker et al., 2012]

Scoring Metrics

First,

- Let $\mathcal{P}$ be the set of true target locations in $\mathbf{x}$
- Let $\hat{\mathbf{x}}_s$ contain all the elements of $|\hat{\mathbf{x}}|$ sorted in increasing order so that the detection threshold $D_{th} = \hat{\mathbf{x}}_s \left(\lceil P_{fa} n) \rceil \right)$

Define two scoring metrics, one familiar one novel:

1. Probability of Detection:

$$P_d = \text{frac} \left(|\hat{\mathbf{x}} \left(\mathcal{P} \right)| \geq D_{th} \right)$$

2. Detection quantile is the P_{fa} required to achieve a perfect P_d:

$$Q_d = \text{frac}(|\hat{\mathbf{x}}| > \min(|\hat{\mathbf{x}}(\mathcal{P})|))$$

   If $Q_d = 0$ it implies that the true target position was the highest-amplitude bin in the estimate.

- Here $\text{frac}(\mathbf{v}) = {}^{\text{sum}(\mathbf{v})}/_{n}$ where $\mathbf{v}$ is a $n \times 1$ boolean vector

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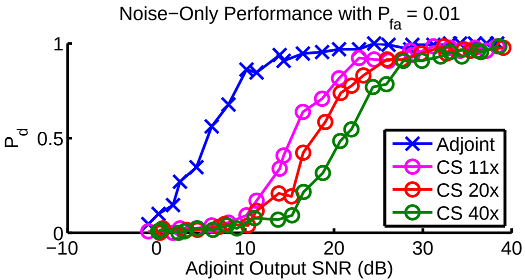
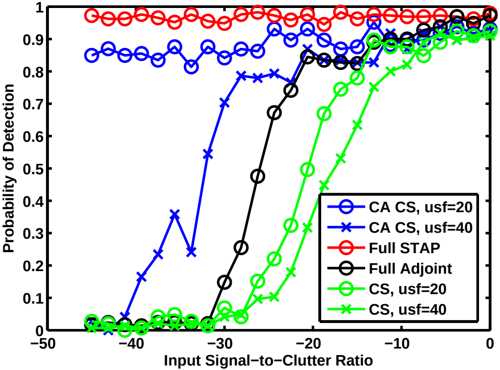
Compressed Sensing in Noise



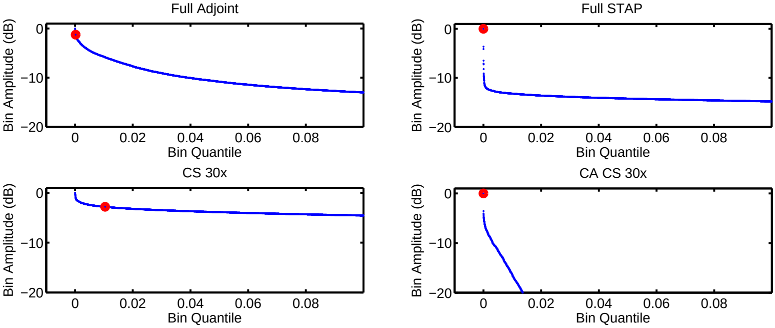
Compressed sensing detection performance degrades with subsampling rate. Each additional octave of under-sampling results in a raising of the noise floor by a factor of two, or 3 dB.

Compressed Sensing in Clutter



The CA CS method improves the probability of detection over the CS case and the fully-sampled matched filter case that does not use the covariance information. These results are shown with a probability of false alarm of .01.

Compressed Sensing in Clutter



A comparison of solution methods applied to a problem with input SNR of 0 dB, input SCR of -20 dB. These plots show the relative amplitude of all the bins in the estimate produced by the identified technique.

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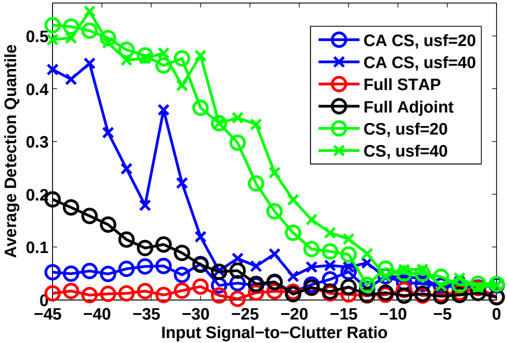

These results show that again, the fully-sampled STAP estimate performs better than all other techniques. Also, the $20\times$ under-sampled CA CS estimate is shown to consistently achieve performance near that of the fully-sampled STAP and better than the fully-sampled adjoint.

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Continuing Work

1. Test against jammer returns
2. Specify selection method for CA CS balancing parameter
3. Estimate covariance matrix from compressed measurements
4. Develop bounds that describe the number of measurements required for a given sparsity level, signal quality, and performance specification
5. Customize transmitted waveforms and measurement operations using prior knowledge of target locations
6. Describe the hardware required to collect the types of measurements required in a CS receiver

Summary

- Performance of compressed sensing algorithms in the presense of realistic correlated interference sources like clutter and jammers has not been well-characterized
- By using interference covariance information we show that significant performance gains can be made relative to standard compressed sensing solutions and relative to the matched filter solution
- Significant work remains to make these techniques operationally useful

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